SERVICEABILITY ASSESSMENT OF THREE LIVELY FOOTBRIDGES IN REYKJAVIK

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Summary

This paper presents results from experimental study on human-induced vibrations of three lively footbridges in Reykjavik. The project was funded by the Icelandic Public Roads Administration with three main focus areas; validating the FE-models used at the design stage in terms of dynamic characteristics, measure the response of the bridges to human induced excitation such as walking, running and jumping and define a set of design guidelines for future implementation regarding vibration serviceability of footbridges in Iceland.

The first bridge is a continuous 86 m long post-tensioned concrete bridge in five spans (max. 23.5m), the second bridge is 169 m continuous post-tensioned concrete bridge in eight spans (max. span 27.1m). The third bridge is a 54 m long steel bridge with two equal spans. The measured vertical acceleration induced by single pedestrians was compared against the predictions and it was found that all the bridges satisfy the criteria set in this standard, i.e. both the calculated and the measured response from single pedestrians were within the acceleration limits.

For a single runner and small groups of people the response exceeds the comfort criteria threshold on all the bridges. Regardless of that, the local authorities have not knowingly received any complaints regarding the serviceability of the bridges so far. The aim is to define suitable design criteria regarding the vibration serviceability for footbridges in Iceland by using the knowledge of existing structures with a known dynamic behaviour to set reasonable acceleration limits where the function and the proposed use of the bridge define the desired performance.

Keywords: dynamics, vibration, design criteria, design competition, response, FE-model updating.

1. Introduction

In 2005 a major six lane urban highway was constructed in the city centre of Reykjavik which required three new footbridges. A design competition which included proposing a new system of pedestrian routes in the area along with the three bridges was held and a proposal by Línuhönnun Consulting Engineers and Studio Granda Architechts was selected. All three bridges have similar cross sections and overall appearance, but different lengths and layout. The design of the bridges is easily adaptable and it is possible that more bridges with the same concept will be built in the future which was indeed part of the brief for the design competition. In 2007, Studio Granda Architechts were nominated for the Icelandic Architectural Awards for the design of the three footbridges over Hringbraut and Njardargata in Reykjavik.

At the design stage it was estimated that two of the bridges could be susceptible to vibrations induced by walking and running due to their low natural frequencies. Based on current code procedures in the Eurocode prestandard, ENV 1992-2 [1] and simplified formulas as provided by Bachmann et al [2], it was estimated that vibration level on the bridges would be tolerable and with a low probability of adverse comments from users.

The codified procedure for serviceability assessment of pedestrian bridges consists of a deterministic single pedestrian load model and a binary pass-fall criterion for the acceptable peak acceleration. This procedure has several shortcomings such as it excludes any randomness in the load and pedestrian pacing rate distributions. Furthermore, it does not take into account the probability of certain crowd morphologies or the expected load intensity on the bridge. As such, bridges in rural settings are treated equally as those in urban areas.

Fig. 1 Footbridge across Hringbraut near Landspítali in Reykjavik

The aim of this study was to define a suitable design criteria regarding vibration behaviour due to human-induced loads on footbridges in Iceland and to confirm predicted behaviour of the new footbridges in Reykjavik. The Icelandic Public Roads Administration funded a research project where those issues were addressed and the results were presented in a detailed report [3]. The project, which was initiated in April 2006 and completed in September 2007 had the following primary objectives:

1. Verify the FE-model used at the design stage (natural frequencies and damping)
2. Compare measured and predicted response from single pedestrians and joggers
3. Measure the response from groups of pedestrians and joggers on different footbridges in Reykjavik
4. Define a set of design guidelines for assessment of human induced vibrations of footbridges for future implementation in Icelandic National Application Documents for the Eurocodes.

The results presented herewith are part of work done in 2006. In 2007 further experiments were conducted as part of this study in collaboration with the Department of Civil Engineering at Technical University of Denmark and the Vibration Engineering Section at University of Sheffield in the UK.

In this paper the measured dynamic properties and measured responses for two of the bridges are described. Results from FE-modelling using a commercially available finite element program are presented and compared to the measured values and to the acceleration limit criteria currently used.

2. Footbridge across Hringbraut near Njardargata (Bridge A)

2.1 Structural configuration of the bridge

The footbridge across Hringbraut near Njardargata is located near the city centre of Reykjavik creating a pedestrian link over a new 6 lane urban highway from the University of Iceland campus to the city centre. The bridge has a spiral shape and is 170 m long in 8 spans with the longest span of 27.1 m as can be seen in Fig 3. The bridge is a continuous post-tensioned beam without any expansion joints and the maximum depth of the beam is 700 mm, i.e. span to depth ratio of 39. The thickness at the edge of the deck is only 170 mm. The width of the bridge deck is 3.2m and a cross-section can be seen in Fig.2. The bridge superstructure is supported on guided elastomeric bearings at the end abutments, providing resistance to lateral forces but free to move in the longitudinal direction at both ends as the spiral shape allows for the thermal movements of the 170 m long bridge. Also at the end of the longest span, spiralling steps diverge from the bridge deck to the ground to shorten the walking distance for pedestrians. The columns are circular stainless steel sections (D=500 mm) filled with concrete to provide resistance to vehicle impact loading and the columns have a fixed moment connection to the bridge deck. Four of the columns are supported on piles but the rest if founded on a compacted fill.

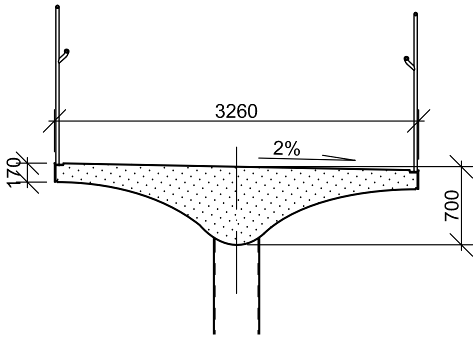

Fig. 2 Footbridge over Hringbraut near Njardargata - Schematic cross section

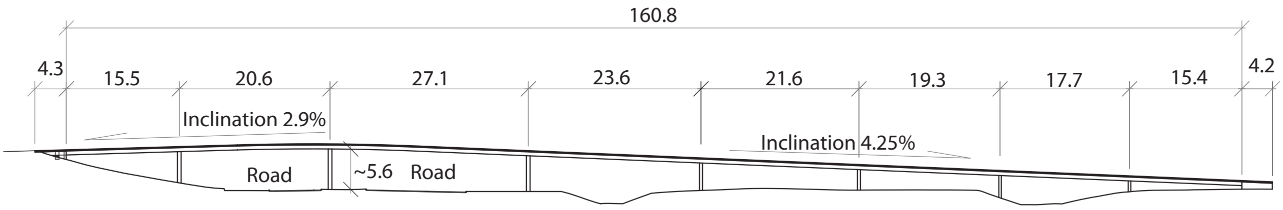

Fig. 3 Footbridge over Hringbraut near Njardargata - Schematic view of bridge elevation

2.2 Dynamic properties

The bridge was modelled using the commercially available finite element program SAP2000 from Computers and Structures Inc. in Berkeley. Beam elements were used to model the bridge except except for the steps in the middle of the bridge where shell elements were used. The bridge deck was modelled with 0.1 m long beam elements, the density of the nodes allowed for the step length to be adjusted with that accuracy.

Response calculations based on a FE-model of a bridge are heavily dependant on an accurate description of both the load and the bridge characteristics, i.e. mass, stiffness and damping which can vary considerably between projects [4]. The main parameters that affect the mode shapes and natural frequencies of footbridges are the boundary conditions, material properties (mass and stiffness) and the influence of parapets and prestressing cables on the stiffness properties of the bridge deck. Other factors such as temperature may also influence the dynamic properties of the bridge. It may be difficult to match the real properties of the bridge using only estimated values of the aforementioned parameters and therefore a modal identification and subsequent FE-model updating is often preferred [5].

At the design stage the footbridge across Hringbraut near Njardargata (Bridge A) was modelled differently. The analysis ignored the effect of prestressing cables and parapets on the stiffness of the deck and also used code prescribed values (Eurocode 2) for the compression strength and Youngs modulus (E). The three dimensional FE-model predicted a fundamental vertical vibration mode with a frequency of 1.92 Hz.

When the vibration serviceability assessment was conducted, the input parameters were updated so that it would reflect as accurately as possible the as-built structure. It included updating the Young's modulus and the deck stiffness by including the influence of prestressing cables, reinforcement and an L-angle stainless steel section cast-in at the edge of the deck which forms part of the parapets. The prescribed type of concrete for the structure was C35/45 with a characteristic compressive strength of f_{ck} = 35 MPa. Results from testing of the concrete cylinders taken from the concrete mix used for the bridge deck resulted in an average compressive strength of $f_{measured}$=65 MPa which is significantly higher than the specified characteristic compressive strength.

Based on calculation methods given in FIB-publication from 1999 [6], the short-term Youngs modulus for a 2 year old concrete with 65 MPa compressive cylinder strength is about 45000 MPa, compared to the value 33500 MPa as used at the design stage according to Eurocode.

The second moment of area, I, for the bridge deck was also updated to reflect the varying position of the prestressing cables and the cast-in part of the parapets. This resulted in a 19% higher value, for a cross section over the support and a 14% higher value for a cross section at midspan compared to a concrete-only cross section.

According to the updated FE-model, the frequency of the fundamental vertical mode increased to $f_{1,est}$ = 2.34 Hz. Figure 4 shows the shape of the first vertical mode based on the updated FE-model.

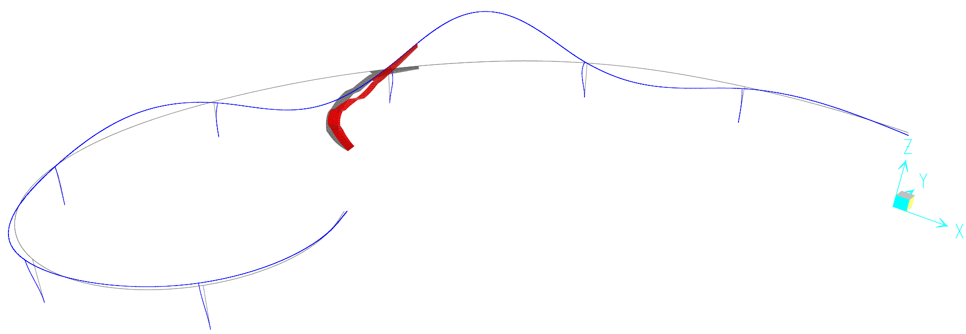

Fig. 4 The first vertical mode for bridge A based on FE-model (f_1=2.34 Hz)

The calibrated FE-model was verified in the field where the fundamental vibration mode was identified to have a frequency f_1 = 2.30 Hz. During the same experiments, the damping ratio was determined as approximately 0.6% of critical.

2.3 Response

The acceleration response due to pedestrians walking across the main span was measured on three locations on the longest span of the bridge. The recorded acceleration time history was low-pass filtered with a cut-off frequency of 5 Hz to exclude noise and high frequency response.

For a single pedestrian, 24 different time series were measured for various step frequencies and the measured peak response ranged from 0.11 – 0.42 m/s^2 depending on the walking frequency. For two pedestrians the peak acceleration response was in the range of 0.19-0.71 m/s^2. The response was also determined for running and for a single runner the peak response was 0.18 – 0.69 m/s^2 and for two runners running in parallel, 0.27-1.43 m/s^2. In Fig.5 the response for single pedestrian/runner can be seen and the response for two pedestrians/runners can also be seen in Fig. 5 as function of the step frequency. The scatter in the measured peak response is a consequence of the randomness in the load, i.e. different people will produce different response, but also the same person will not produce the same response in two consecutive crossings. This observation is in broad agreement with earlier observations of the load induced by people walking, [4, 7, 8].

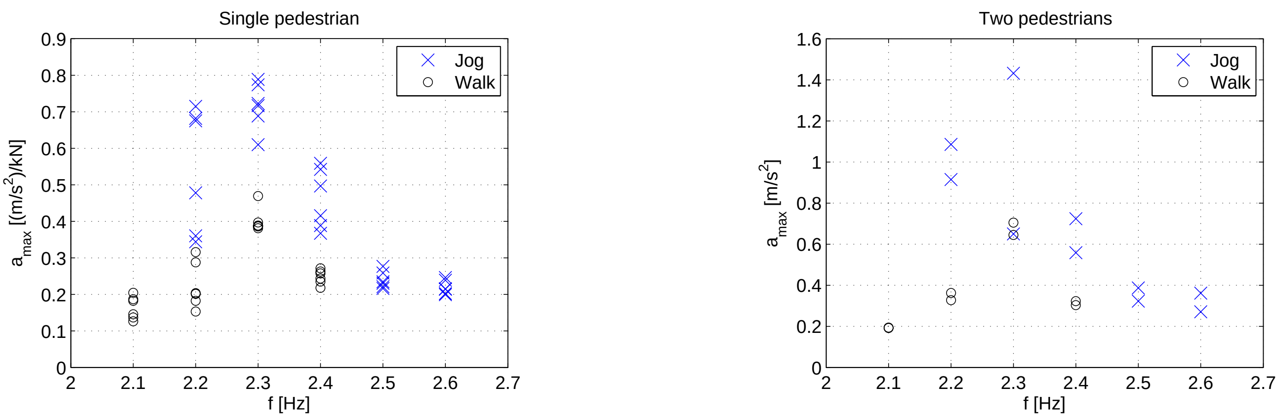

Fig. 5 Peak measured vertical response for a single pedestrian and a single runner (left). Peak measured vertical response for 2 pedestrians and 2 runners (right). The response is calibrated with the weight of the participants.

A group test was also conducted with 25 people participating. Two experiments were made where the group walked briskly across the bridge (approximate step frequency 2.3 Hz) resulted in a peak response of 0.89 m/s^2. Two series for slow running were also measured (approx. 2.3 Hz) and two series for normal running (2.5 Hz). The peak vertical acceleration was 1.03 m/s^2. The measured time series for fast walk and slow running can be seen in Fig. 6 together with the running 1 sec. RMS value of the acceleration.

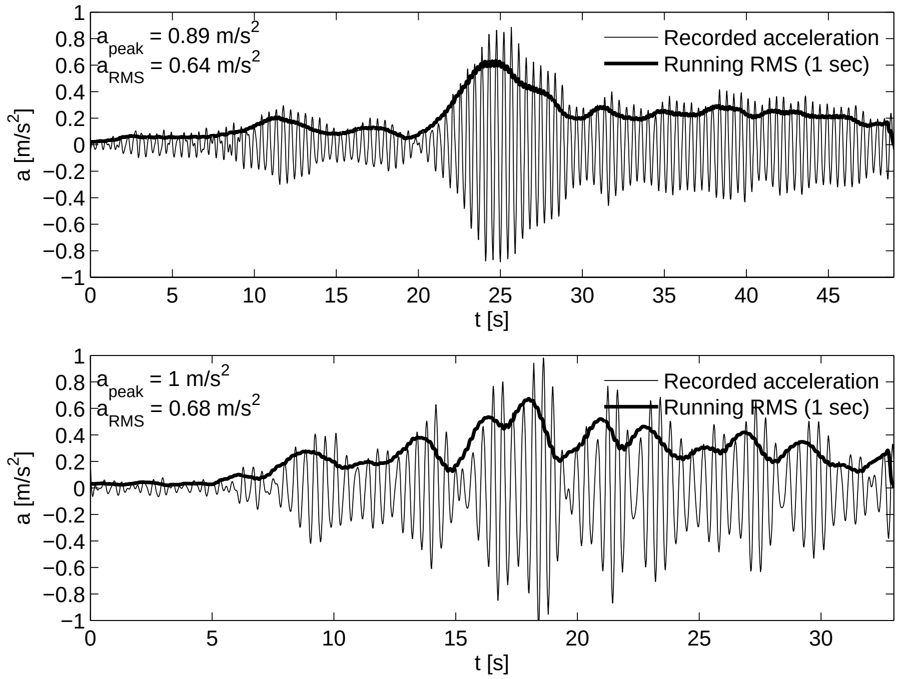

Fig. 6 Measured time series for a group(25 people) walking (above) and running (below) across the bridge.

In Fig. 7 the response of the bridge for different loadcases can be seen and the response compared with the current design criteria based the Eurocode prestandard, [1], where the peak acceleration criteria is defined as $a_{peak} = 0{,}5\sqrt{f_o}$ for a single walking pedestrian where f_0 is the fundamental frequency of the bridge. For this particular bridge, the criterion becomes $a_{peak} < 0.76$ m/s^2. It should be noted, that according to a new draft of the ISO 10137 standard [9] the acceleration should generally not exceed a response factor R= 60 or R=30 if one or more pedestrians standing still have to be accounted for. The response factor is defined as the ratio between the RMS acceleration response and the base curve for human perception to whole body vibration as defined in [10]. For vibration at the frequency 2.30 Hz, the base acceleration is 0.0078 m/s^2 and the acceleration criteria become 0.235 m/s^2 (R=30) and 0.47 m/s^2 (R=60). Both these criteria are stricter than the one in the Eurocode and they are exceeded both during the single pedestrian and group experiments.

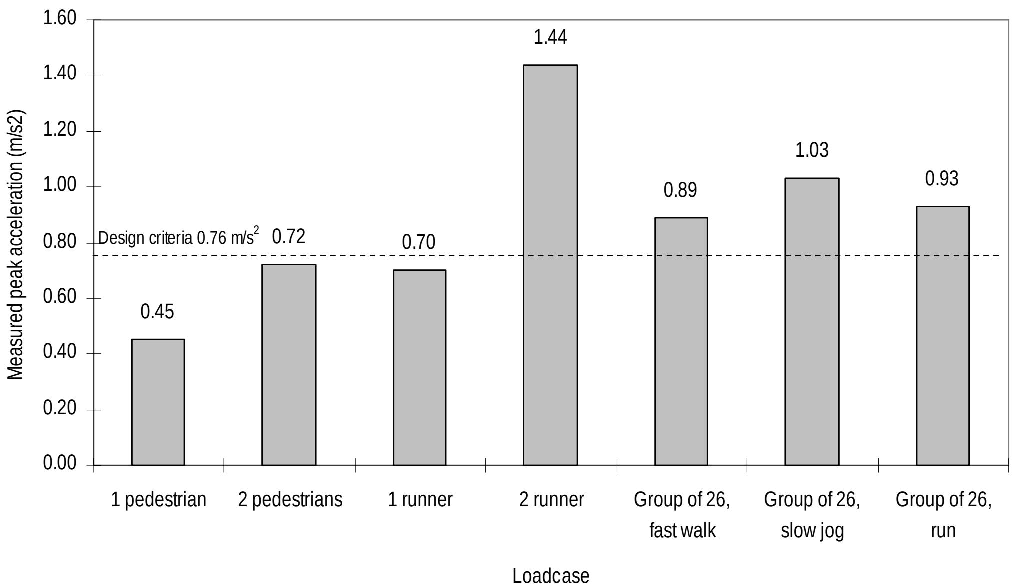

Fig. 7 Vertical acceleration response for different loadcases. Peak acceleration values for a given loadcase.

3. Footbridge across Hringbraut near Landspitali (Bridge B)

3.1 Structural configuration of the bridge

The bridge is on a footpath linking a popular outdoor leisure area to the city centre of Reykjavik and is close to the University Hospital which is the largest hospital in Iceland. The bridge is 86 m long in 5 spans with the longest span of 23.5 m, see Fig.1. The bridge is curved in plan, starting with a 60m radius curve but narrows down to a 30m radius over the longest span. The bridge has the same structural form as the footbridge in the previous chapter, a post-tensioned continuous beam with a maximum depth of 700 mm, i.e. span to depth ratio of 34. To simplify the construction, the three bridges which were part of the project, were all designed with the same structural configuration but with different levels of post-tensioning. The bridge superstructure is supported on elastomeric bearings, fixed both laterally and longitudinally at the north end but only laterally at the south end. The columns are circular stainless steel sections (D=500 mm) filled with concrete as for the previous bridge but are all founded on compacted fill. Fig. 8 shows an elevated view of the bridge and the cross section is shown in Fig 2.

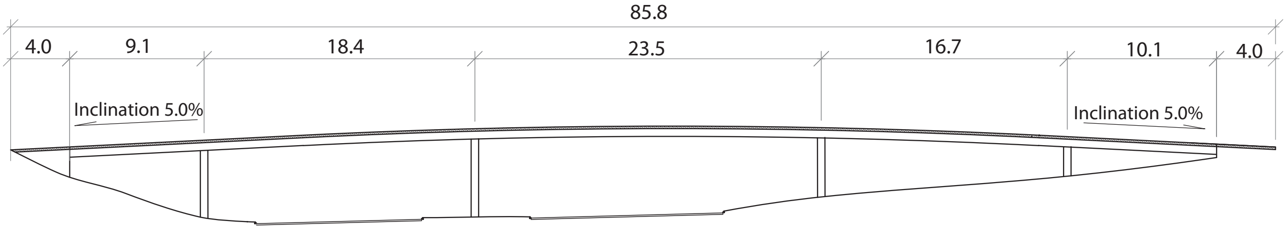

Fig. 8. Footbridge across Hringbraut near Landspitali in Reykjavik. Elevation

3.2 Dynamic properties

During the initial design, a FE-model of the bridge was constructed using same input parameters as for bridge A. The same approach was taken for calibration of this model as described in the previous chapter for bridge A, i.e. by using the same stiffness parameters. The natural frequency of the fundamental vertical vibration mode was determined experimentally to f_1 = 3.0 Hz, the same as predicted by the FE-model. The damping ratio was determined from 11 different measurements to be ζ = 0.00925 on average, or approximately 0.9% of the critical damping ratio.

3.3 Response

As the frequency of the first vertical mode is 3.0 Hz, excitation from a walking pedestrian is not of interest but runners can easily excite the bridge. Three runners participated in the experiment where each of them ran back and forth across the bridge at different step frequencies controlled by a digital metronome. Each participant performed in total 14 runs at 7 different frequencies in the range 2.6 – 3.2 Hz. The measured response ranged from 0.14 – 1.02 m/s^2 depending on the step frequency. In Fig. 9 the peak acceleration for each time series is shown as function of the step frequency for a single runner. It can be seen that the largest acceleration is at resonance with the frequency of the first vertical mode as expected. The maximum allowable peak acceleration according to the Eurocode [1] is 0.87 m/s^2 and from Fig. 9 it is seen that this limit is exceeded for a single jogger at and around the resonance frequency.

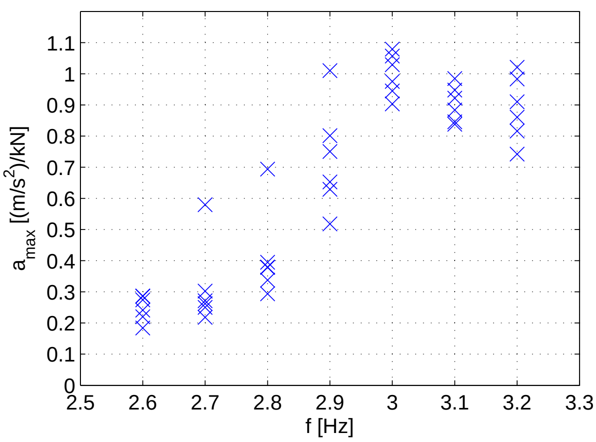

Fig. 9 Peak measured vertical response for a single runner. The response is calibrated with the weight of the runner.

4. Measured response compared to predicted response for bridge A

Several response analyses have been conducted using load models from the literature and the Eurocode with the scope to test the capability of the FE-model to predict the vibrations of the bridges. The load from a single pedestrian was described as a truncated Fourier-series with the first 4 load harmonics taken as a function of the step frequency as reported in [7]. The same load model was also used to describe the load from a running pedestrian except for different dynamic load factors (DLF) and step lengths which were taken from Bachmann [2]. A series of linear time history analyses were conducted for a single pedestrian and a single runner where the step frequency was varied from 1.1-2.8 Hz for walking and 2.0-3.2 Hz for running. The load induced by a single pedestrian can be written as:

$$F(x,t) = G_p\left[1+\sum_{j=1}^{3}\alpha_j \sin(2j\pi f_p t - \phi_j)\right]\delta(x - f_p l_p t) \tag{1}$$

Where $x(t)$ is the location of the load input, l_p is the step length, δ is the Dirac's delta function and f_p is the step frequency. The relationship between the DLF and the step frequency can be seen in Eq. (2a)-(2d) and was taken from Young [7] and relate to the average DLF based on experiments.

$$\alpha_1 = 0.37(f_p - 0.95) \le 0.5, \qquad 1.0 \le f_p \le 2.8Hz \tag{2a}$$
$$\alpha_2 = 0.54 + 0.0088 f_p, \qquad 2.0 \le 2f_p \le 5.6Hz \tag{2b}$$
$$\alpha_3 = 0.026 + 0.0150 f_p, \qquad 3.0 \le 3f_p \le 5.6Hz \tag{2c}$$
$$\alpha_4 = 0.010 + 0.0204 f_p, \qquad 4.0 \le 4f_p \le 11.2Hz \tag{2d}$$

An often used load model to calculate the response of a bridge to a single pedestrian is the one presented originally by Blanchard et.al. [11] and has been implemented in several codes of practice, e.g. the British Standard BS 5400 and the Eurocode. The peak acceleration response can be calculated by using a simple expression:

$$a_{peak} = 4\pi^2 f_0^2 y_s k \alpha \Phi \tag{3}$$

Where f_0 is the frequency of fundamental vertical mode, y_s is the static deflection to a force 700N, α is the DLF at the step frequency f_0 and k and Φ are configuration and response factors respectively, [2]. According to the Eurocode, the DLF should be taken as 0.257 independent of the step frequency.

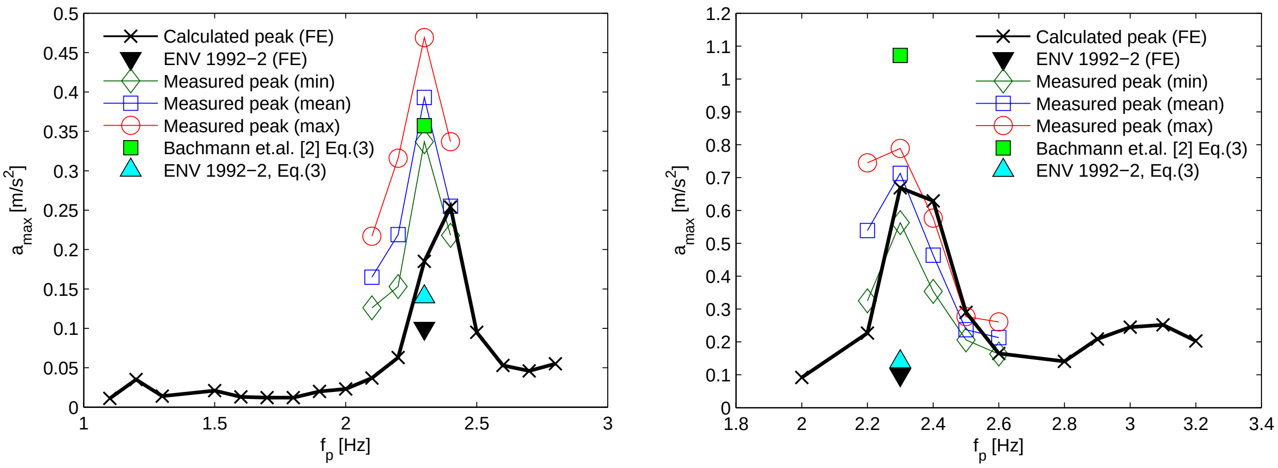

Fig. 11 Comparison between measured acceleration and predicted acceleration by FE-model for bridge A depending on the step frequency. For a single pedestrian (left) and single runner (right).

In Fig. 11 an example of the comparison between the predicted peak acceleration and the measured acceleration can be seen for a single pedestrian and a single runner depending on the frequency of the excitation. It can be seen that the response predicted by the FE-model is a bit lower than the measured values for a single pedestrian which can probably be explained by the DLFs used in the Fourier-series describing the applied loads. It can also be seen that the simplified formulae given by Bachmann [2], Eq. (3) agrees well with the measured response. The predicted response for a single runner agrees well with the measured response. Another reason for the mismatch between the calculated and measured response may be attributed to the FE model and that the modal updating was based on only a rough fit to the fundamental vertical mode. Therefore a considerable uncertainty in the modal mass and the effect of higher vibration modes must be accounted for. However, when using realistic values for the bending stiffness of the bridge deck (instead of using code specified values) in a FE framework, a reasonable initial estimate can be obtained for the liveliness of the bridge when applying the load model in Eq. (1) and (2) and including all vibration modes likely to be excited by the force. This can be done be applying the force directly onto the model where the both the load amplitude and the position changes with time, or the modal response can be calculated directly for each vibration mode and subsequently combined linearly in time using standard modal analysis.

5. Discussion and Conclusions

Vibration measurements and a serviceability assessment have been conducted on four different footbridges in Iceland. Three are part of this study (only two presented in this paper) but previously a footbridge across Miklubraut near Raudagerdi had been studied. In that bridge, dampers were installed after complaints had been observed. That bridge was located near a sports centre and was frequently crossed by groups of joggers resulting in large amplitude vibrations. During the aforementioned serviceability assessment the measured peak acceleration from 2 runners was 3.7 m/s^2 [12]. As a consequence, tuned mass dampers were installed to suppress the vibrations. Summary of the measured response for the bridges in Reykjavik can be seen in Table 1 below.

Table 1. Summary of measured vibration for 4 different footbridges in Reykjavik

Footbridge	Measured Vertical Frequency	Measured response				Criteria from Eurocode [1]
		1 pedestrian	2 pedestrians	1 runner	2 runners	a_{max}
Hringbraut near Njardargata	2.3 Hz	0.45 m/s^2	0.72 m/s^2	0.70 m/s^2	1.44 m/s^2	0.76 m/s^2
Hringbraut near Landspitali	3.0 Hz	-	-	1.02 m/s^2	1.71 m/s^2	0.87 m/s^2
Miklabraut near Grundargerdi [3]	2.7 – 2.8 Hz	0.67 m/s^2	-	3.76 m/s^2	-	0.82 m/s^2
Miklabraut near Raudagerdi [12]	2.7 Hz	0.26 m/s^2	0.58 m/s^2	0.95 m/s^2	3.66 m/s^2	0.82 m/s^2

The three footbridges considered in this study can all be classified as lively but still satisfy the current code criteria. Despite this, the bridges are not considered problematic as no complaints have been received regarding the vibration behaviour. However for group loads or loads from running people the limit criteria is exceeded.

Currently there are no footbridges in Reykjavik which have significant daily traffic of pedestrians which require particular attention to group loadings. Attention needs however to be paid to specific events as can be seen on the footbridge across Hringbraut near Landspitali (Bridge B) in figure 12. During one of the measurements, the bridge was packed with people in a parade as part of a summer festival for teenagers in Reykjavik but the measured peak acceleration during the 10 minutes it took the parade to cross, was lower than for one runner as the fundamental vertical frequency of that bridge is 3.0 Hz and the bridge is not likely to be excited excessively by a slowly moving crowd.

Fig. 12 Footbridge across Hringbraut near Landspitali. A parade crossing the bridge.

Based on this study, a work is in progress to define suitable design criteria that is likely to be used as a basis for the Icelandic national annex regarding footbridge vibration. The aim is to help the owner of the structure under consideration to define a suitable serviceability criteria regarding vibration due to pedestrians or runners. The criteria will be depend on the location of the bridge, the importance of the footpath or if any specific events (e.g. rare crowd events) are expected to occur. Similar approach has been proposed by e.g. MacKenzie et al [13] or in the French SETRA guidelines [14].

The importance of using an up-to-date knowledge will be emphasized both on the loading input, the modelling of the structure and the acceleration limit criteria. It will also be recommended to take account for the possible installation of dampers at a later stage if vibration serviceability will be a problem. This will apply to structures which at the design stage are expected to be susceptible to vibration. The installation of dampers afterwards would consequently be more economic as no major changes would be needed to the structure and it could fit within the aesthetics of the bridge as well.

Recent research has emphasised the importance of using a probabilistic approach to vibration serviceability to take into account the randomness in the load, the response and the human response to vibration, see e.g. [15-17]. A probabilistic approach could provide a platform for a new generation of code procedures that will supersede current deterministic load models and binary pass-fall acceleration criterion. This approach would encourage a rationale discussion between the owner and the designer to quantify the expectations to the behaviour of the structure and to define acceleration limits in terms of probability of exceedance rather than as a fixed number.

5.1 Acknowledgements

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